THE BROOKHAVEN-COLUMBIA PLASMA LENS*)

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SUMMARY

A uniform current density plasma is generated by discharging a high-voltage capacitor bank into two electrodes in a low pressure gas. The magnetic field within the plasma produces achromatic focusing of pions and kaons travelling along the electrode axis.

I. INTRODUCTION

A spark-chamber study of neutrino interactions is being carried out at the AGS by a joint Columbia-BNL group. The beam of muon neutrinos is produced from the decay of pions and kaons coming from a beryllium target in a 30 GeV external proton beam:

$$p + \text{Be} \rightarrow \text{nuclear products} + \pi\text{'s and/or K's} \qquad (1)$$

$$\pi^{\pm} \text{ or } K^{\pm} \rightarrow \mu^{\pm} + \{\nu/\bar{\nu}\} \ . \qquad (2)$$

Since the direction of the neutrino differs only slightly from that of the parent meson, a focusing device which decreases the intrinsic divergence of the secondary meson beam will increase the neutrino flux at the detector. (The spark chambers subtend a half angle of 20 milliradians; the useful flux at 4 GeV/c extends to about 100 milliradian.) Conventional magnetic focusing devices (quadrupoles) are unsuitable because of the simultaneous requirements of a large momentum bandwidth and a large angular aperture. In addition, there is no practical way to selectively focus mesons of one sign and thereby permit the study of neutrino and antineutrino reactions separately.

At Brookhaven we have attempted to apply an old idea first used by Panofsky in 1948 to focus protons from the 184-in cyclotron[1]). This is that a cylindrical, longitudinal column of current generates an aximuthal magnetic field which increases linearly with the radius. (The gaussian circle encloses more current as r^2, but increases in circumference only linearly in r.) In high-energy physics applications the currents required are so large that it constitutes a plasma discharge and profits from the plasma technology that has recently been developed. Its appeal (aside from its being "different") lies in the absence of the hole at small angles (present in the CERN magnetic horn) and the possibility of control over the radial current distribution.

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II. LENS OPTICS

The magnetic field produced by a uniform current density J of radius R along the z axis is:

$$B_\Theta = \frac{J}{5}\frac{r}{R^2}\ (r \le R)\ . \qquad (3)$$

The equations of motion of a particle in this field are:

$$\ddot{r} + k^2 r = 0\ , \qquad (4a)$$

where $\dot{r} = dr/dz$ and

$$k^2 = \frac{60\pi J}{p}$$

J is the density in amps/cm^2, p is momentum in e.v.

For a point target at the origin, the solution is:

$$r = A \sin kz \qquad (4b)$$

$$A = \frac{\Theta_0}{k}\ ,$$

where Θ_0 is the maximum entrance angle contained by the lens.

Particles of one sign will be focused parallel to the axis when $kz = \pi/2$; particles of the other sign will be defocused since k is imaginary.

The radius R and length L of the column are determined by the condition:

$$R = \frac{\Theta_0}{k} \sin kL = \frac{\Theta_0}{k} \text{ for } kL = \pi/2\ . \qquad (5)$$

Therefore, the current required to form a parallel beam of all particles of momentum p_0 emitted into the lens at an angle $\Theta < \Theta_0$ is:

$$I = \frac{p_0 \Theta_0^2}{60}\ , \qquad (6)$$

for example, $p_0 = 3$ GeV and $\Theta_0 = 6°$, $I = 5 \times 10^5$ amperes.

The length and diameter of the discharge is determined by the acceptance required:

$$\Theta_0 = \frac{\pi D}{2L}$$

and the practical consideration that D and L must be large compared to the target dimensions.

The momentum dispersion is from Eq. (4)

$$\Theta_{final} = \Theta_{initial} \cos \frac{\pi}{2}\sqrt{p_0/p}\ , \qquad (7)$$

and is seen to vary very slowly with p. The meson gain in density is given by:

$$\left(\frac{\Theta_o}{\Theta_f}\right)^2 \quad \text{weighted by the} \quad \frac{d^2N}{d\Omega dp} = g(\Theta_i, p) .$$

(The actual gain, of course, depends upon the detector dimensions.)

In practice, the experimentally determined pion and kaon flux are used in a Monte Carlo programme which traces trajectories through the magnetic field distribution measured in the plasma discharge.

III. ENGINEERING DESIGN

The lens consists of a large fused silica tube with stainless steel electrodes inserted in each end. The downstream electrode is connected by a metal tube, concentric with the lens, to a header at the upstream end. This produces a neat coaxial layout and permits cooling water to circulate between the ground return and the insulated tube. The electrodes are also water cooled. The lens is continuously evacuated by a rotary vacuum pump; the gas pressure is adjusted by a needle valve in the supply line. A block diagram of the main components is shown in Fig. 1.

1. Main storage bank

The main bank consists of 33 identical modules containing a capacitor, discharge and crowbar ignitrons. The total capacitance is 480 microfarads, nominal maximum voltage 20 kV. In practice the operating level is 16 - 18 kV. Modules are designed to be quickly replaced in the bank if necessary, any number can be operated in parallel without interaction as they are isolated from the charging source and discharge trigger generator. Each module is connected to the load header by two coaxial cables, B.I.C.C. type 20P2. Relevant electrical characteristics of the main bank are given in Table 1. A simplified schematic is shown in Fig. 2. When the discharge ignitrons are triggered, the current in the lens begins to increase in an approximately sinusoidal manner (only approximately, because the parameters of the load vary with time).

When the voltage on the storage bank reverses polarity, the crowbar ignitrons are fired automatically and the reverse voltage excursion is limited: this produces a worthwhile increase in capacitor life. The trigger generator consists of a five-ohm pulse forming network discharged by a 7390 thyratron. Each ignitor circuit contains a pulse inverting and isolating transformer, and a series hundred-ohm resistor for proper load sharing. The total load presented to the generator roughly matches the pulse-forming network. Thus a rectangular trigger pulse is generated. The duration is four microseconds, rise time half a microsecond and maximum magnitude is 15 kV. The voltage on the ignitor is typically 4 kV during normal operation. The energy supplied to the ignitor must be a compromise; too much causes early ignitor burn-out, too little may result in tardy ignitrons failing to fire at all.

The main bank is charged at a constant current, about 4.5 amps, supplied by a three-phase monocyclic network, step-up transformer and rectifier bank. Each module is connected via a high-vacuum diode to the common charging point. Thus, the failure of a module, for example, due to premature ignitron breakdown, does not cause the rest of the bank to be dumped into the defective module. When the desired charging level is reached a tetrode connected in shunt at the charging supply output is driven full on, the charging current is diverted and the output voltage falls to a low value. The bank remains charged as the series diodes become non-conducting. The bank begins to recharge when the tetrode is turned off by an AGS timing pulse.

The lens is operated at the same repetition rate as the AGS, about 1 pulse every 2 seconds. Hence component life expectancy must be of the order of 10^6 pulses so that the device has reasonable reliability. The ignitrons used to discharge the main bank were life tested to over a million pulses at a fifty percent higher current than encountered with the lens as a load. The storage capacitors are not allowed to develop reverse voltages of any magnitude during the discharge. To date only one capacitor has failed.

2. B_z supply

The axial magnetic field is provided by an air-cooled solenoid wrapped around the lens. The flux density generated by this coil is about 4 kG. As the problem of heat dissipation makes a dc coil impractical, field is also generated by discharging a storage bank. To avoid large eddy currents in the lens ground return and electrodes, the discharge frequency is relatively low, about one hundred cycles per second. At this frequency the discharge ignitrons extinguish automatically at the end of a conducting cycle, thus the bank is left with some stored energy at the end of the discharge. The main bank is fired at peak current during the first half cycle of the B_z discharge. A simplified circuit of the supply is shown in Fig. 3. The important circuit parameters are included in Table 1.

IV. DISCUSSION OF THE PLASMA DISCHARGE

The plasma lens consists of an insulating tube filled with a gas such as argon at a pressure of between 10 and 1,000 microns, through which is discharged a large high-voltage capacitor bank. With these gas conditions and flat-faced electrodes, the initial breakdown is along the walls of the tube. This current heats and ionizes the gas near the walls of the tube so that it becomes a reasonably good annular conductor, with a thickness (skin depth) significantly smaller than the radius. When the current is sufficiently large that the inward magnetic pressure, $B_\Theta^2/8\pi = I^2/200\pi r^2$ (gauss, amps, cm), exceeds the outward pressure of the contained gas and trapped axial magnetic field, the current sheath begins to collapse inwards. This radial motion of the current is an example of the dynamic pinch effect, the theory of which was given by Rosenbluth, Garwin and Rosenbluth in 1954²⁾. The approximation is made that the gas becomes an infinitely good conductor soon after breakdown and before very much current is being conducted. In this case, the collapsing tube of current sweeps all the gas before it, always in a thin shell of increasing mass. The equation of motion of the snow-plough model, as this interpretation is appropriately called, is derived by equating the net inward pressure to the increase in radial momentum of the collapsing shell of gas:

$$B_o^2/8\pi - \left(B_z^2/8\pi + nkT\right) = \frac{d}{dt}\left[\frac{\rho_o}{2r}\,(r_o^2 - r^2)dr/dt\right], \tag{8}$$

where ρ_o is the initial gas density and r_o is the initial radius. The solution of this equation for field gradients of 50 to 100 V/cm, ringing frequency of about 20 kc/s, and argon gas pressure of order 100 microns gives pinch velocities of order 1 cm/μsec, i.e., a pinch time about equal to the quarter-cycle time of the capacitor bank. Further analysis requires consideration of the properties of real gases. Most important is the finite resistivity which gives the current sheath a non-zero thickness which increases with time. It is precisely this increase in thickness occurring during the pinch cycle which gives the possibility of a uniform current density. If the pinch velocity is very high, the size of the cylinder of uniform current density will be very small, resulting in a small angular aperture (but high centre momentum); if the pinch velocity is very low, the total current will have decayed to a low value by the time the uniform current density is obtained. The externally applied axial magnetic field which exerts an outward pressure of $B_z^2/8\pi$, and the gas pressure which determines the inertia of the current sheath can be seen from the snow-plough equation to give mechanisms for controlling the pinch velocity. Large energy losses are implicit in both pinch-retarding techniques; if the gas pressure is high, the energy goes into the kinetic energy of the gas, if the axial magnetic field is high, the energy goes into compressing the trapped B_z field. In the plasma lens about half the total energy in the capacitor bank is converted into B_Θ field. Figures 4 and 5 show the time development of this magnetic field for various gas pressures and axial magnetic fields. It should be noted that the time over which the magnetic field has the desired spatial distribution is significantly longer than the 2.5 μsec spill time of the external proton beam at the AGS. This extended period of stability is due primarily to the relatively large axial magnetic field which, in resisting compression, also stabilizes the system against sausage and kink instabilities.

V. EXTERNAL CIRCUIT PARAMETERS OF THE PLASMA DISCHARGE

Because the current distribution in the lens changes markedly during the discharge period, the load inductance and resistance are both functions of time and an analytic solution to the equation is difficult, if not impossible, to obtain. As the lens is used only for a brief period near the time of maximum current, it is important to know the peak value and the time to reach it. In a series R-L-C circuit with inductance varying with time, the describing equation is:

$$\frac{d^2 i}{dt^2} + \frac{di}{dt}\left(\frac{R}{L(t)} + \frac{2d[L(t)]}{dt}\,\frac{1}{L(t)}\right) + \frac{i}{L(t)C} = 0 \tag{9}$$

Expand L(t) as a series and truncate:

$$\begin{aligned} L(t) &= L + K_1 t + K_2\, t^2 + \ldots. \\ &\approx L + K_1\, t \end{aligned}$$

then Eq. (9) becomes:

$$\frac{d^2 i}{dt^2} + \frac{di}{dt}\left(\frac{R + 2K_1}{L + K_1 t}\right) + \frac{i}{C(L + K_1 t)} = 0 \ , \qquad (10)$$

the constant part of L(t) includes cable and module inductance plus the inductance of the plasma with uniform current distribution. The K_1 term represents the increase in inductance due to stored magnetic energy between the plasma and the ground return as the plasma radius shrinks. By using the expression for the inductance of a coaxial cable and knowing the pinch rate, one can make a close approximation for K_1:

$$K_1 = \frac{\mu_0}{2\pi} \ln\left(\frac{R_0}{R_1}\right) \cdot \frac{1}{t_{R_1}} \ , \qquad (11)$$

the exponent of the damping term in the solution to 'Eq. (10) may be greatly increased due to K_1. An iterative solution to Eq. (10) using these approximations for the inductance gave good agreement with the observed shape and magnitude of the current wave. Similar results were obtained by a computer solution to the snow-plough model equations.

VI. OPERATIONAL USE

The plasma lens as described has been operated successfully for approximately 24 hours of beam time, during which it produced a net gain in neutrino events in the spark chambers of about × 3. In anticipation of the next neutrino run, the power supply is being modified to run at a higher voltage and to produce a higher B_z field. Additional probe measurements are planned to study the details of the loss mechanisms.

ACKNOWLEDGEMENTS

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REFERENCES

1) W. Panofsky and W. Baker, Rev.Sci.Instr., 21, 5 (1950).

2) M. Rosenbluth, R. Garwin and A. Rosenbluth, United States Atomic Energy Commission Report LA-1850.

Table 1

	Main bank	B_z supply
Bank size, μF	480	660
Bank voltage, max.	20 kV	10 kV
Stored energy, Joules, max.	96,000	33,000
Peak current, amps.	5×10^5	
Load inductance	$\approx 120 \times 10^{-9}$ H	
Lead inductance	25×10^{-9} H	negligible
Q	≈ 3	5
Voltage reversal	10%	75%
Repetition period, sec	2	2
Time to I_{pk}	$\approx 15\mu$ sec	$\approx$ 2.2 msec

FIGURE CAPTIONS

Figure 1 : System block diagram.
Figure 2 : Simplified circuit of main bank.
Figure 3 : Simplified circuit of B_z supply.
Figure 4 : Field distribution.
Figure 5 : Field distribution.

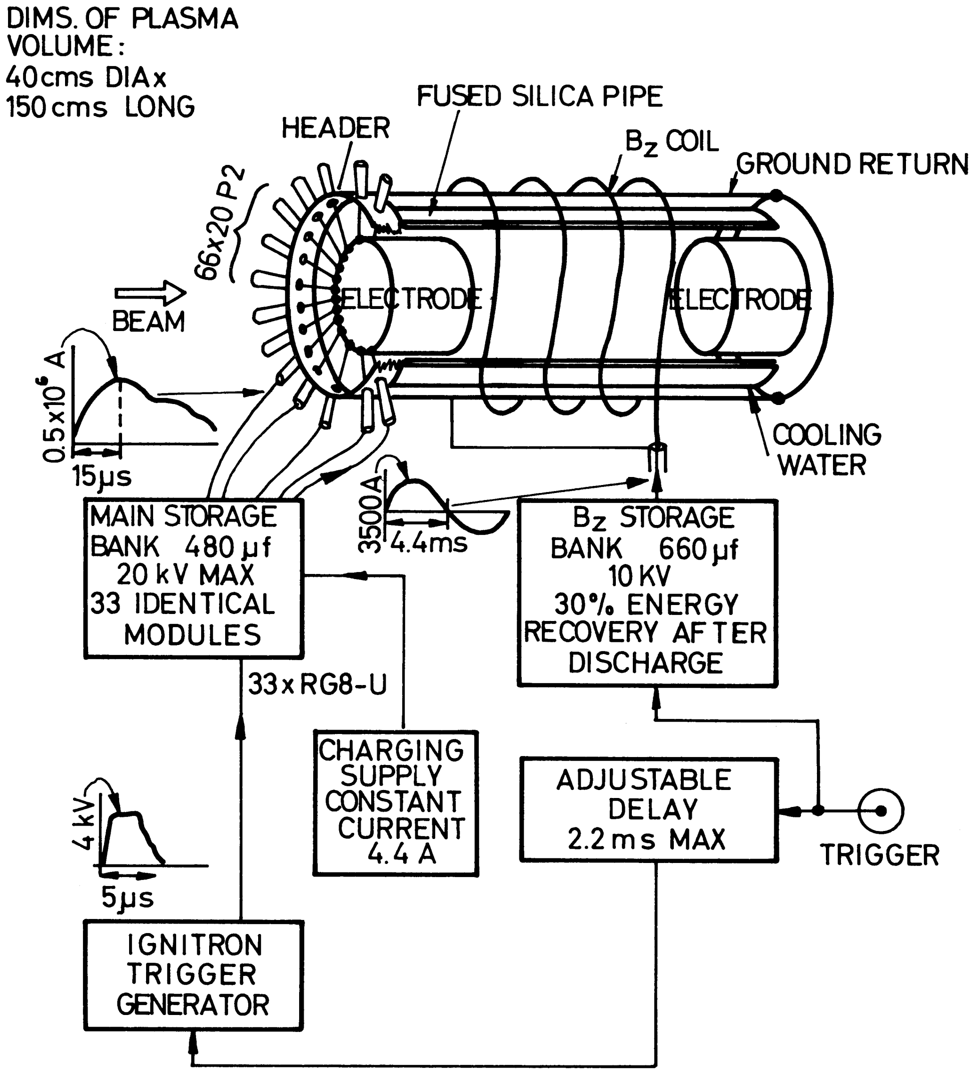
DIMS. OF PLASMA VOLUME: 40cms DIA x 150 cms LONG
FUSED SILICA PIPE
HEADER
B_Z COIL
GROUND RETURN
66x20 P2
BEAM
ELECTRODE
ELECTRODE
0.5×10^6 A
15µs
COOLING WATER
3500A
4.4ms
MAIN STORAGE BANK 480 µf 20 kV MAX 33 IDENTICAL MODULES
B_Z STORAGE BANK 660 µf 10 KV 30% ENERGY RECOVERY AFTER DISCHARGE
33 x RG8-U
CHARGING SUPPLY CONSTANT CURRENT 4.4 A
ADJUSTABLE DELAY 2.2 ms MAX
TRIGGER
4 kV
5µs
IGNITRON TRIGGER GENERATOR

Fig. 1

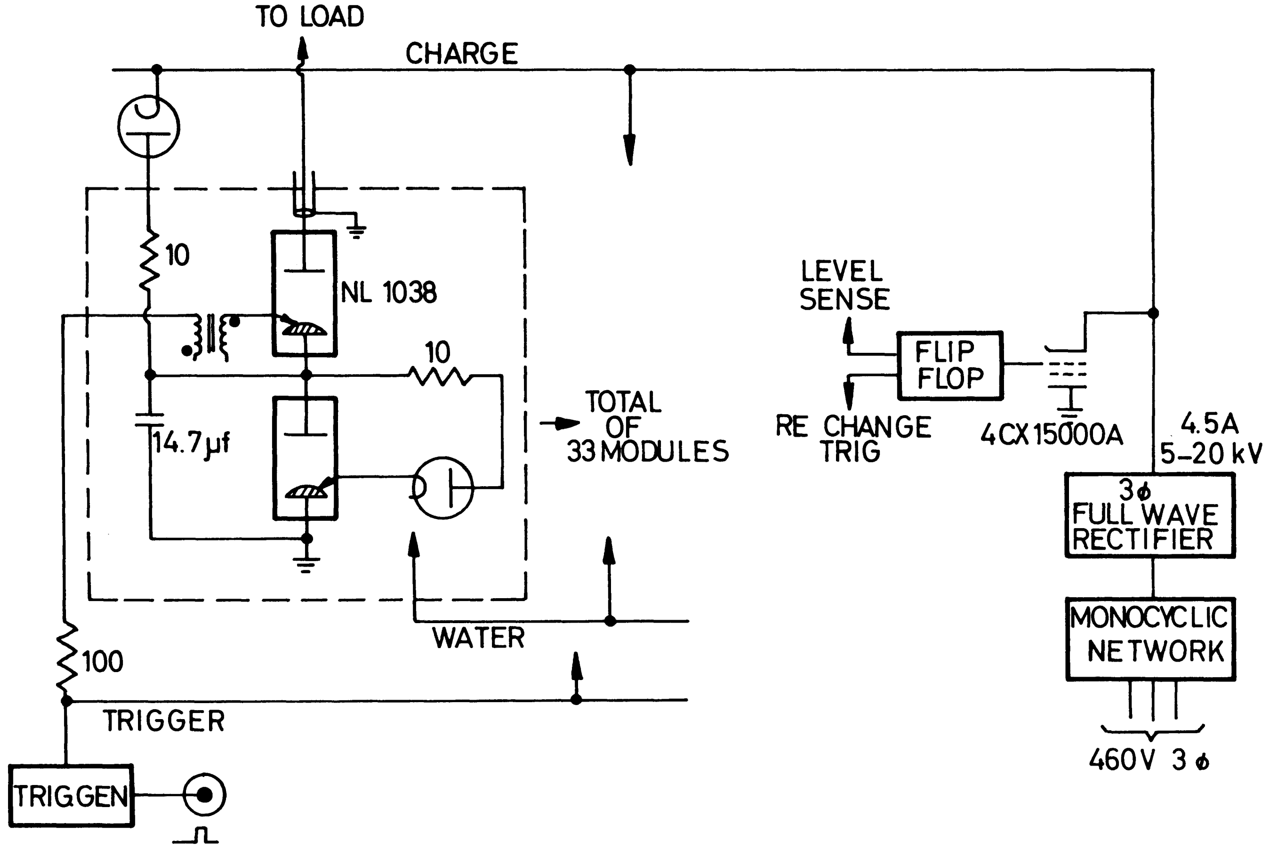
TO LOAD
CHARGE
10
NL 1038
10
14.7μf
TOTAL OF 33 MODULES
WATER
100
TRIGGER
TRIGGEN
LEVEL SENSE
FLIP FLOP
RE CHANGE TRIG
4CX15000A
4.5A
5-20 kV
3φ FULL WAVE RECTIFIER
MONOCYCLIC NETWORK
460V 3φ

Fig. 2

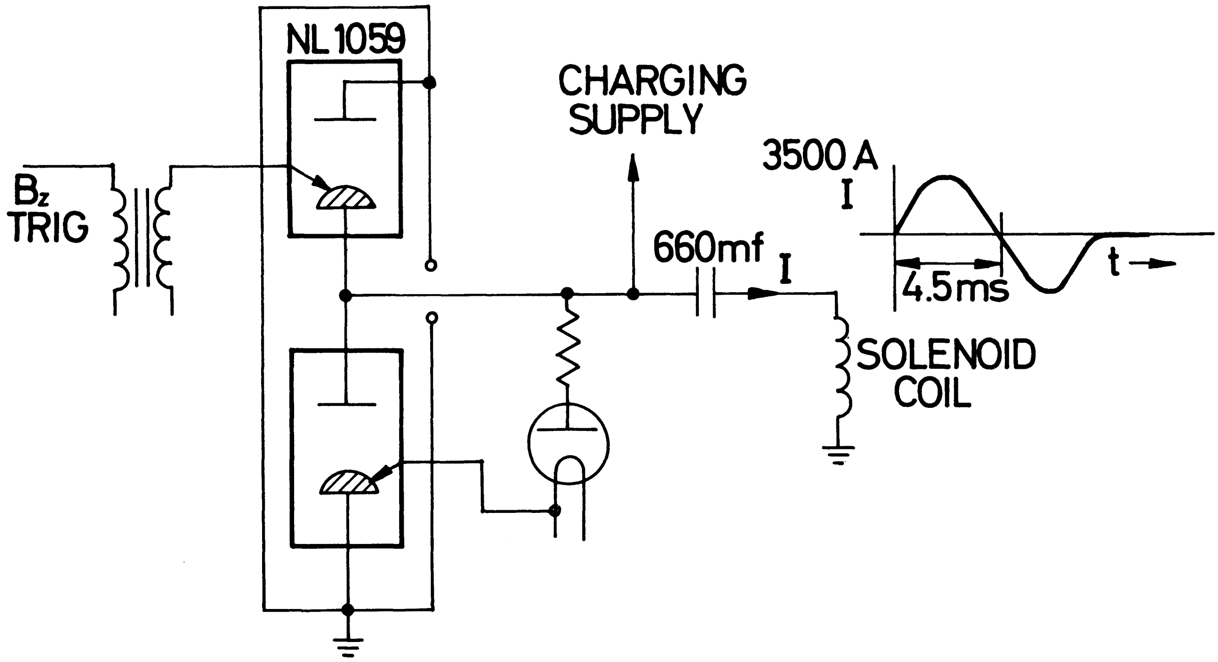
NL1059
CHARGING SUPPLY
3500 A
I
Bz TRIG
660mf
I
t
4.5ms
SOLENOID COIL

Fig. 3

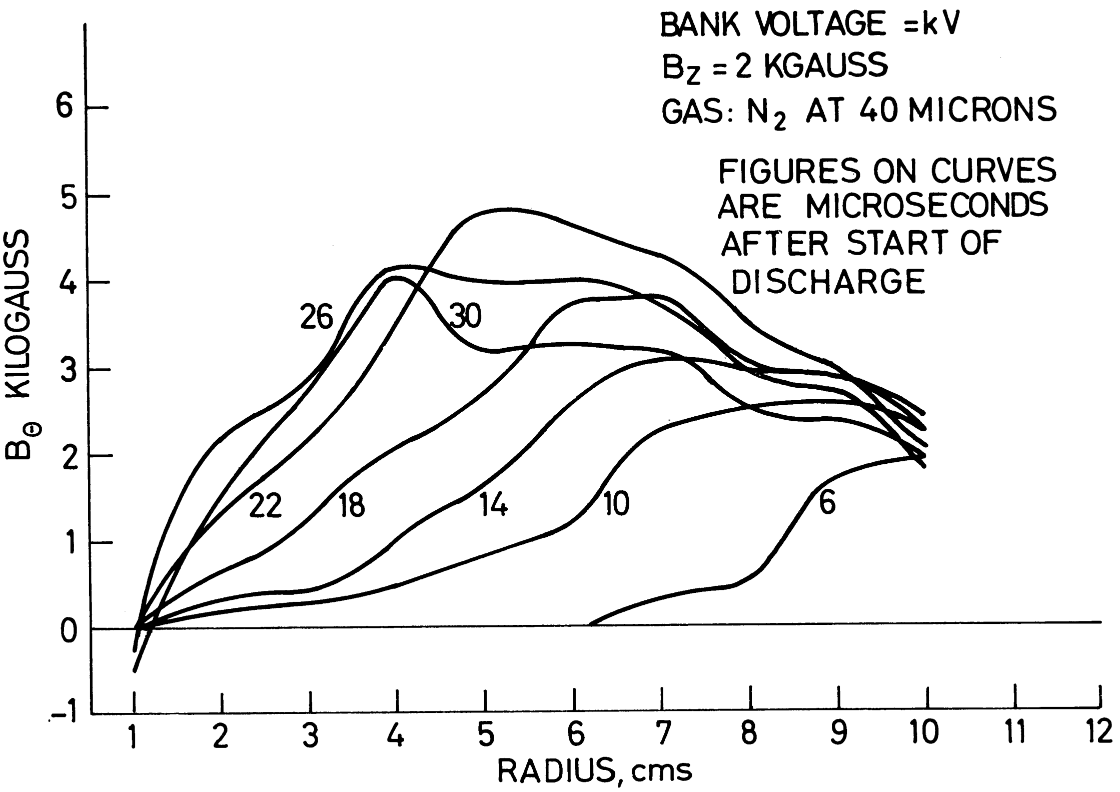
BANK VOLTAGE =kV
BZ = 2 KGAUSS
GAS: N2 AT 40 MICRONS
FIGURES ON CURVES ARE MICROSECONDS AFTER START OF DISCHARGE
Bθ KILOGAUSS
6
5
4
3
2
1
0
-1
1
2
3
4
5
6
7
8
9
10
11
12
RADIUS, cms
26
30
22
18
14
10
6

Fig. 4

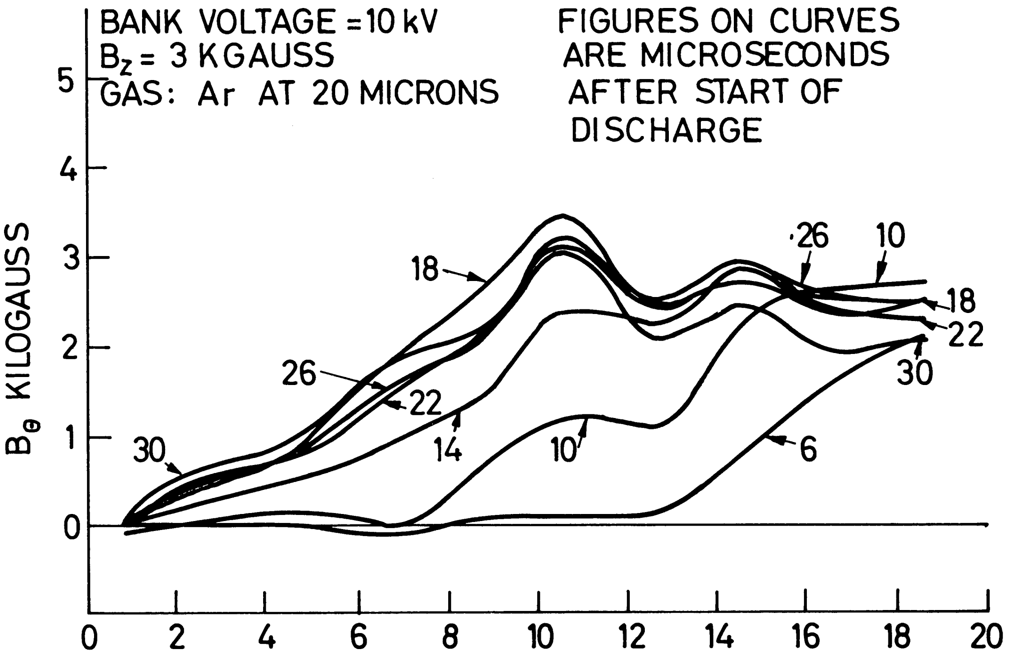
BANK VOLTAGE =10 kV
BZ = 3 KGAUSS
GAS: Ar AT 20 MICRONS
FIGURES ON CURVES ARE MICROSECONDS AFTER START OF DISCHARGE
Bθ KILOGAUSS
5
4
3
2
1
0
0
2
4
6
8
10
12
14
16
18
20
RADIUS IN cms
18
26
22
14
10
30
6
·26
10
18
22
30

Fig. 5